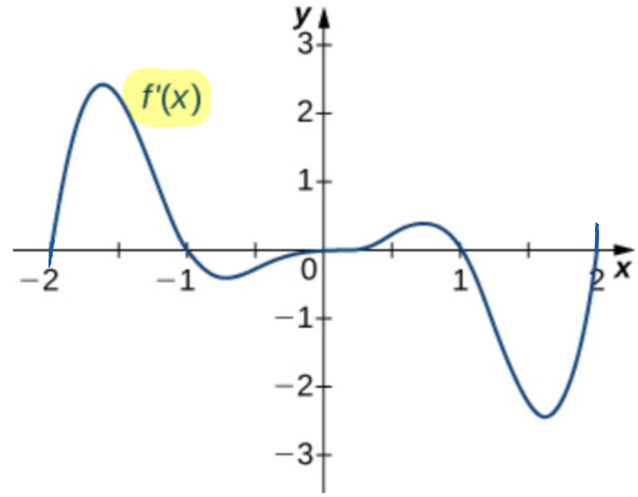
Derivatives & Shape of Graphs:

- Analyze the following graph & find local maxima/minima.



Note: ① • f is increasing whenever $f'(x)>0$
• f is decreasing whenever $f'(x)<0$

② • If $x=a$ is a local maxima, then on the left of $x=a$, $f(x)$ is increasing (ie, $f'(x)>0$) & on the right of $x=a$, $f(x)$ is decreasing (ie, $f'(x)<0$)

• If $x=b$ is a local minima, then on the left of $x=b$, $f(x)$ is decreasing (ie, $f'(x)<0$) & on the right of $x=b$, $f(x)$ is increasing (ie, $f'(x)>0$)

In the given graph, $\begin{cases} f'(x)>0, & \text{when } -2<x<-1 \ \& \ 0<x<1 \\ f'(x)<0, & \text{when } -1<x<0 \ \& \ 1<x<2 \end{cases}$

Since critical points are those where $f'(x)=0$, then we have our critical pts as $x=-2,-1,0,1,2$.

Hence, $x=-2,-1,0,1,2$ are the potential local maxim/minima.

For $x=-2$, on left: $f'(x)<0$ } $\Rightarrow$ local minima
on right: $f'(x)>0$

$x=-1$, on left: $f'(x)>0$ } $\Rightarrow$ local maxima
on right: $f'(x)<0$

$x=0$, on left: $f'(x)<0$ } $\Rightarrow$ local minima
on right: $f'(x)>0$

$x=1$, on left: $f'(x)>0$ } $\Rightarrow$ local maxima
on right: $f'(x)<0$

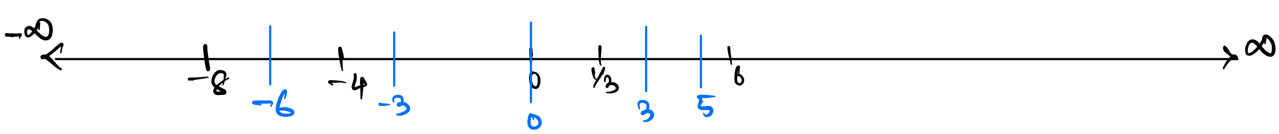
$x=2$, on left: $f'(x)<0$ } $\Rightarrow$ local minima.
on right: $f'(x)>0$

- Suppose $\boxed{f''(x)} = x^2(x-5)^4(x+6)^3(x^2-9)$

  ① How many point of inflections f has?

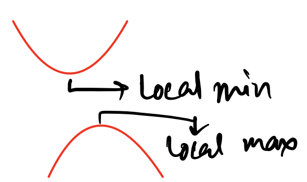
  ② If f has local horizontal tangent at $-8, \frac{1}{3}, 0, 6, -4$, then find the local maxima/minima if exists.

① Now, x^2 & $(x-5)^2$ are allways non-negetive & $f''(x)$ changes its signs only due to the factors $(x+6)^3$ & (x^2-9).



Note: $f''(x)>0 \Rightarrow$ Concave Up

$f''(x)<0 \Rightarrow$ Concave Down



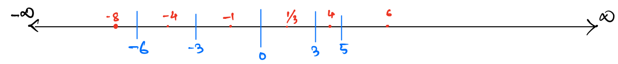
$f''(a)=0$ & $f''(x)$ changes sign at $x=a \Rightarrow$ point of inflection

By taking $f''(x)=0$ we get our potential points of infection

ie, $x^2(x-5)^4(x+6)^3(x^2-9)=0$

$\Rightarrow x^2(x-5)^4(x+6)^3(x+3)(x-3)=0$

$\Rightarrow x=0, 5, -6, -3, 3$



Test for $x=-6$:

pick any value between $(-\infty, -6)$, say $x=-8$

$$f''(-8) = \underbrace{(-8)^2}_{+ve}\underbrace{(-8-5)^4}_{+ve}\underbrace{(-8+6)^3}_{-ve}\underbrace{((-8)^2-9)}_{+ve} < 0$$

pick any value between $(-6, -3)$, say $x=-4$

$$f''(-4) = \underbrace{(-4)^2}_{+ve}\underbrace{(-4-5)^4}_{+ve}\underbrace{(-4+6)^3}_{+ve}\underbrace{((-4)^2-9)}_{+ve} > 0$$

Hence, $x=-6$ is an inflection point.

Test for $x=-3$:

pick any value between $(-6, -3)$, say $x=-4$

$$f''(-4) = \underbrace{(-4)^2}_{+ve}\underbrace{(-4-5)^4}_{+ve}\underbrace{(-4+6)^3}_{+ve}\underbrace{((-4)^2-9)}_{+ve} > 0$$

pick any value between $(-3, 0)$, say $x=-1$

$$f''(-1) = \underbrace{(-1)^2}_{+ve}\underbrace{(-1-5)^4}_{+ve}\underbrace{(-1+6)^3}_{+ve}\underbrace{((-1)^2-9)}_{-ve} < 0$$

Hence, $x=-3$ is an inflection point.

Test for $x=0$:

pick any value between $(-3, 0)$, say $x=-1$

$$f''(-1) = \underbrace{(-1)^2}_{+ve}\underbrace{(-1-5)^4}_{+ve}\underbrace{(-1+6)^3}_{+ve}\underbrace{((-1)^2-9)}_{-ve} < 0$$

pick any value between $(0, 3)$, say $x = 1/3$

$$f''(1/3) = \underbrace{(1/3)^2}_{+ve}\underbrace{(1/3-5)^4}_{+ve}\underbrace{(1/3+6)^3}_{+ve}\underbrace{((1/3)^2-9)}_{-ve} < 0$$

Hence, $x = 0$ is an inflection point.

Test for $x = 3$:

pick any value between $(0, 3)$, say $x = 1/3$

$$f''(1/3) = \underbrace{(1/3)^2}_{+ve}\underbrace{(1/3-5)^4}_{+ve}\underbrace{(1/3+6)^3}_{+ve}\underbrace{((1/3)^2-9)}_{-ve} < 0$$

pick any value between $(3, 5)$, say $x = 4$

$$f''(4) = \underbrace{(4)^2}_{+ve}\underbrace{(4-5)^4}_{+ve}\underbrace{(4+6)^3}_{+ve}\underbrace{((4)^2-9)}_{+ve} > 0$$

Hence, $x = 3$ is an inflection point.

Test for $x = 5$:

pick any value between $(3, 5)$, say $x = 4$

$$f''(4) = \underbrace{(4)^2}_{+ve}\underbrace{(4-5)^4}_{+ve}\underbrace{(4+6)^3}_{+ve}\underbrace{((4)^2-9)}_{+ve} > 0$$

pick any value between $(5, 9)$, say $x = 6$

$$f''(6) = \underbrace{(6)^2}_{+ve}\underbrace{(6-5)^4}_{+ve}\underbrace{(6+6)^3}_{+ve}\underbrace{((6)^2-9)}_{+ve} > 0$$

Hence, $x = 5$ is not an inflection point.

② Given f has local horizontal tangents, ie, extremum points at $x = -8, \frac{1}{3}, 0, 6, -4$

From above analysis: $f''(-8) < 0 \Rightarrow$ concave down
ie, local maxima.

$f''(-4) > 0 \Rightarrow$ concave up
ie, local minima.

$f''(0) = 0 \Rightarrow$ point of infection
but not maxima/minima.

$f''(1/3) < 0 \Rightarrow$ concave down
ie, local maxima.

$f''(6) > 0 \Rightarrow$ concave up
ie, local minima.